

# Steps in model-based trajectory searching as a tool for biped design

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## Summary

We aim to explore model-based motion planning as a tool to aid in development of a three-link walking machine. The physical prototype is designed with regards to an existing dynamic model which ensures a rich variety of feasible walking gaits. This approach puts the emphasis of gait generation on motion planning instead of control action, allowing for more accurate analysis of such effects as perturbations of parameters, forces arising from unmodeled parts of dynamics and adding energy storing elements such as springs.

## Introduction & Motivation

Motion planning for underactuated walking robots is a challenging task that has attracted wide attention since McGeer's seminal paper on planar passive walkers on shallow slopes [1]. McGeer's design would later be improved upon by Collins et al. [2] when the first three-dimensional, kneed passive walker was built, creating a much more human-like device. The energy efficiency of these passive gaits showed that producing gaits for underactuated walking machines was not only feasible, but also highly desirable. Specifically, the existence of passive gaits meant that actuation could be seen as a correction of deviations from the energy efficient gait, rather than the main generator of motion. A well known benchmark task of a two link compass-gait biped is to investigate the existence of passive or low power gaits, but its limitation of consisting solely of two legs does not allow for investigation of the dynamics of an upper body and its influence on the gait existence and control. Feasible gaits for such robots with a torso, as seen in Figure 1, have previously been found using a model-based approach [3], even for a robot with two degrees of underactuation [4].

We propose a similar approach to search for gaits for a walking machine with underactuation one, with a focus on verification of model-based motion planning as an accurate and beneficial tool for generating gaits for physical walking machines. Utilizing the proposed method, numerous energy efficient walking gaits have been discovered for the three-link biped, but the results are so far constrained to simulations. With the introduction of the physical prototype, the following topics can be investigated: 1) verification of various hypotheses found in model-based simulations, including the possibility to induce energy efficient locomotion; and 2) a greater un-

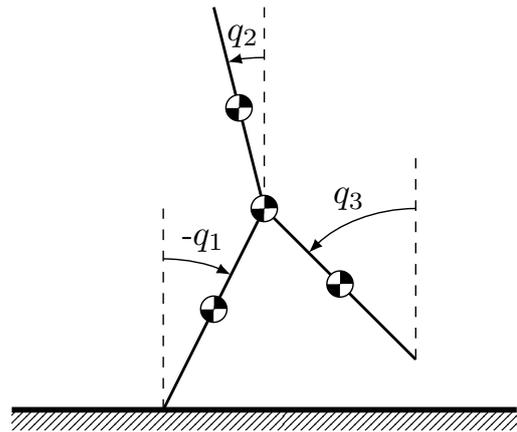


Figure 1: Schematic drawing with coordinate convention for the three-link biped.

derstanding of the dynamics of impact and its role in stabilization of gaits.

## Robot Description

The robot is constrained to move in the sagittal plane, and is equipped with two feet per leg to maintain balance in the coronal plane. In order for the feet of the swing leg to clear the ground when swinging past the stance leg, the robot is equipped with retractable point feet controlled by servos. Each of the legs are actuated by a low-power Maxon DC-Motor combined with an 1:6 gearing ratio gearbox relative to the upper body at the hip joint of the robot. The rotor of each motor is connected to the axis of rotation of a corresponding leg, and then further attached to the other leg via a ball bearing due to overlapping mechanical structure to achieve symmetry of the overall design. As a result the torso is attached to the legs via the stator of each motor at the hip.

The robot is equipped with an encoder on each of the motors, meaning that the encoders measure relative angles between each of the legs and the torso. In addition, the absolute angle of the torso is measured using an on-board Inertial Measurement Unit (IMU). Together these three sensors yield absolute angle and angular velocity information for each link.

Measurement and motor control is currently handled by external dSPACE hardware connected to a PC. The mathematical model and control algorithm codes are developed and uploaded to the real-time platform using

MATLAB Simulink. It is planned as future work to implement both the model and the control algorithm on-board using a BeagleBone Black open hardware Linux computer.

## Motion Planning and Stabilization

Given the dynamical equations of the system,

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu, \quad (1)$$

the virtual holonomic constraints approach is utilized to reduce the dimensionality of the problem, and thus formulating an optimization problem with a finite number of decision variables. The search algorithm looks for suitable relations on the form  $q = \Phi(P, \theta) \in \mathbb{R}^3$  between the generalized coordinates, such that the reduced dynamics [5]

$$\alpha(\theta)\ddot{\theta} + \beta(\theta)\dot{\theta}^2 + \gamma(\theta) = 0, \quad (2)$$

parametrized by the scalar motion generator  $\theta(t)$ , admits a periodic solution when coupled with the discontinuous impact dynamics occurring at the end of each walking step. These constraints are not an intrinsic part of the dynamics of the system, but rather relations between the degrees of freedom imposed by an external controller.

Using Bézier curves to represent these constraints with parameters  $P$  for  $\theta \in [0, 1]$ , a search is initialized with variables  $(P, \kappa, \dot{\theta}_+)$ , where some of the parameters of  $P$  are predetermined by the initial and final link configuration over a step, the impact velocity map and the torque continuity directly at each iteration of the search. The remaining parameters,  $P^*$ , are found by solving the following non-linear optimization problem:

$$\underset{(P^*, \kappa, \dot{\theta}_+)}{\text{minimize}} \int_0^1 \left\| \Phi'(\tau)^T Bu(\tau) \right\| d\tau, \quad \Phi'(\theta) := \frac{d}{d\theta} \Phi(\theta),$$

subject to

$$(\alpha(1)^2 \kappa^2 - \alpha(0)^2) \dot{\theta}_+^2 + 2 \int_0^1 \alpha(\tau) \gamma(\tau) d\tau = 0.$$

An orbitally stabilizing controller is generated by stabilizing the linearized dynamics of a set of 5 transverse coordinates. To this end, a LQR is utilized, which is found by solving the time-varying Riccati differential equation with one jump.

## Preliminary Results and Discussion

1. The model-based motion planning algorithm has produced numerous gaits that satisfy the physical constraints on torque that the physical prototype requires.
2. The novelty of the structure of the search allows for vast customization of parameters such as lean angle of torso, step length and walking speed.
3. Several very energy efficient gaits have been found for smaller step lengths (10-20 cm), where the energy loss due to impact is reduced. This energy

efficiency may be further improved with different curve representations of the virtual holonomic constraints, suggesting the possibility of increasing the degree of underactuation and still achieve locomotion. The search for energetic efficiency may therefore be a constructive way towards discovering pure horizontal passive gaits. Note that truly passive walking gaits on level ground have been found for similar models [6].

4. All gaits have been verified in simulation and found to be sufficient to guarantee walking behaviour in the simulated version of the physical robot. They are, however, all dependent on the choice of impact model, which is currently formulated using the principle of conservation of angular momentum. The properties of the true physical impact might differ from those modeled with regards to duration, magnitude of forces and compliance of materials. There might also be a considerable difference in how well the gaits respond to the discrepancy, leading to interesting insights into the design of robust gaits and several iteration of biped design.

There are still some redesign and assembly that is required before the physical biped can take its first steps and proper verification of the gaits found, and by extension the application of model-based trajectory search, can be undertaken. Currently the robot is undergoing system identification experiments to accurately determine several key physical parameters such as centers of mass, moments of inertia and friction in drive trains and bearings.

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